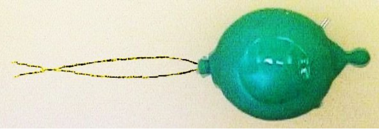
Elastic Tail Propulsion at Low Reynolds Number

An Experimental Study



Tony S. Yu

Advisor: Prof. A.E. Hosoi

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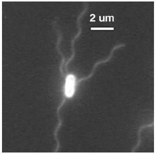
Department of Mechanical Engineering
Massachusetts Institute of Technology

HATSOPOULOS MICROFLUIDS LABORATORY

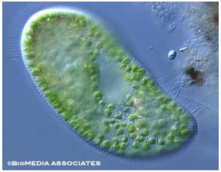
Motivation

Swimming at low Reynolds number → swimming at small scales

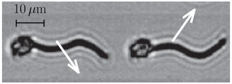
- Understanding biological systems (Fundamental Science)
  - Small → microorganism.
  - How do mechanics affect biological structures?



L. Turner, W.S. Ryu, H.C. Berg (2000)



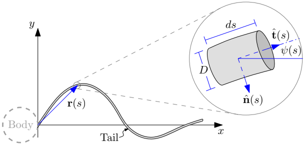
- Mechanical design (Engineering)
  - Optimal swimming motion
  - Applications:
    - Targeted drug delivery.
    - Minimally invasive surgery.
    - Navigable *in situ* sensing.

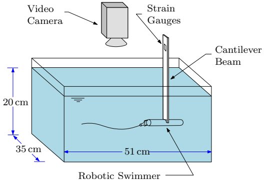


Dreyfus, *et.al* (2005)

Outline

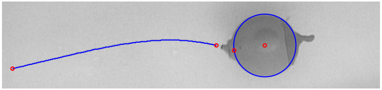
- Dynamics of Elastic Tails in Viscous Fluids
  - Nonlinear
  - Linear





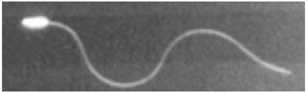
- Fixed Swimmer Experiments
  - Stall Force
  - Tail Shapes

- Free Swimmer Experiments
  - Swimming Velocity
  - Tail Shapes



Swimming at Low Reynolds Number

Reynolds number: $\boxed{\mathrm{Re} = \dfrac{\text{inertial forces}}{\text{viscous forces}} = \dfrac{\rho V L}{\mu}}$



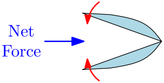
C.J. Brokaw

gallery.hd.org

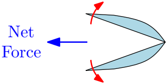
Re $\ll 1$: fluid motion governed by Stokes' equations

$$\nabla p = \mu \nabla^2 \mathbf{u} \qquad \text{and} \qquad \nabla \cdot \mathbf{u} = 0$$

Scallop Theorem: reciprocal motion produces no net force at low Reynolds number.



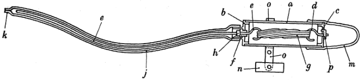
Closing Cycle



Opening Cycle

Life at Low Reynolds Numbers

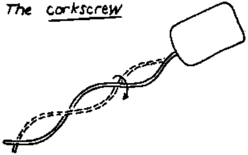
- G.I. Taylor (1951,1952)
- Sir James Lighthill (1975)
- Stephen Childress (1981)
- E.M. Purcell (1976)

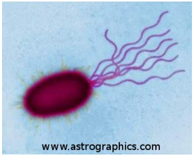


Taylor (1952)

Swimmers from Purcell's *Life at Low Reynolds Numbers*

- Rotating Helix Swimmer
- Three-Link Swimmer
- Flexible Oar Swimmer





Investigators

- E.M. Purcell (1997)
- M. Kim, J.C. Bird, A.J. Van Parys, K.S. Breuer, and T.R. Powers (2003)
- H.C. Berg and L. Turner
- and many, many others

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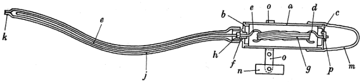

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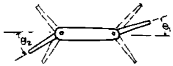

Investigators

- L.E. Becker, S.A. Koehler, and H.A. Stone (2002)
- B. Chan and A.E. Hosoi
- D.S.W. Tam and A.E. Hosoi (2007)

Life at Low Reynolds Numbers

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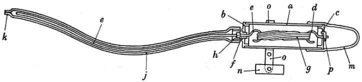

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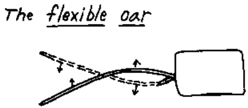
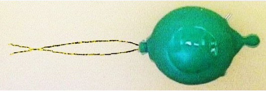


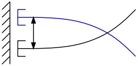
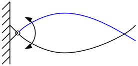
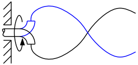


Investigators

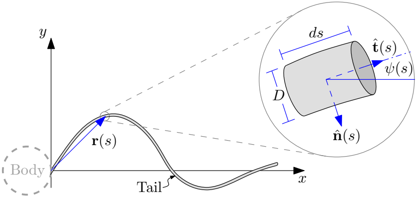
- K.E. Machin (1958)
- C.H. Wiggins and R.E. Goldstein (1998)

- M.C. Lagomarsino and C.P. Lowe (2000)
- T.S. Yu, E. Lauga, and A.E. Hosoi (2006)

Investigations in Elastic Tail Swimming

Actuation	Experiment	Numerics	Linear Theory	Full Theory
Transverse Oscillation 	Wiggins, *et al.* (1998)	Lowe (2003) and Lagomarsino (2004)	Wiggins & Goldstein (1998)	Camalet & Jülicher (2000)
Angular Oscillation 	Yu, Lauga, & Hosoi (2006)	Lowe (2003) and Lagomarsino (2004)	Machin (1958) and Wiggins & Goldstein (1998)	Camalet & Jülicher (2000)
Rotation 	Koehler & Powers (2000)	Lagomarsino (2004)	—	Wolgemuth (2000)

Dynamics of Elastic Tails *



- s – arclength
- $\hat{\mathbf{n}}$ – inward pointing normal
- $\hat{\mathbf{t}}$ – unit tangent vector
- τ – local tension
- ψ – local angle
- subscripts s and t denote derivatives

Viscous Forces

Resistive-force theory

$$\mathbf{f}_d = -[\xi_\perp \hat{\mathbf{n}}\hat{\mathbf{n}} + \xi_\parallel \hat{\mathbf{t}}\hat{\mathbf{t}}] \cdot \mathbf{r}_t$$

- $\xi_\perp$ – transverse ($\hat{\mathbf{n}}$) drag coefficient
- $\xi_\parallel$ – axial ($\hat{\mathbf{t}}$) drag coefficient
- $\mathbf{r}_t$ – local velocity

Elastic Forces

Elastic force from elastic energy

$$\mathcal{E} = \int_0^L \left[\frac{A}{2}{\psi_s}^2 + \frac{\Lambda}{2}{\mathbf{r}_s}^2\right] ds, \quad \mathbf{f}_\mathcal{E} = -\frac{\delta \mathcal{E}}{\delta \mathbf{r}}$$

$$\mathbf{f}_\mathcal{E} = -(A\psi_{sss} - \psi_s \tau)\hat{\mathbf{n}} + (A\psi_{ss}\psi_s + \tau_s)\hat{\mathbf{t}}$$

- $A = EI$ – bending stiffness of tail

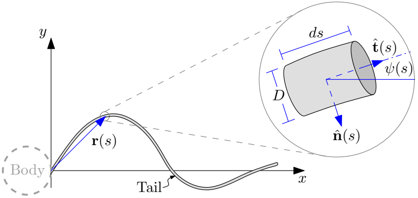
Local equilibrium requires: $\mathbf{f}_d + \mathbf{f}_\mathcal{E} = 0 \quad \longrightarrow$ coupled, nonlinear PDEs

$$\psi_t = -\frac{1}{\xi_\perp}\left(A\psi_{ssss} - \tau\psi_{ss} - \tau_s\psi_s\right) + \frac{1}{\xi_\parallel}\left(A\psi_s^2\psi_{ss} + \tau_s\psi_s\right)$$

$$\tau_{ss} - \frac{\xi_\parallel}{\xi_\perp}\tau\psi_s^2 = -A\left(1 + \frac{\xi_\parallel}{\xi_\perp}\right)(\psi_s\psi_{sss}) - A\psi_{ss}^2$$

* Camalet & Jülicher, *New J. Phys.*, 2000

Dynamics of Elastic Tails *



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Viscous Forces	Elastic Forces
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Viscous Forces

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* Camalet & Jülicher, *New J. Phys.*, 2000

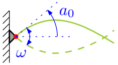
Linearized Dynamics of Elastic Tails †

For small deflections (*i.e.*, $\psi \ll 1$ such that $\psi \approx y_x$)

Linear Equation of Motion:

$$\boxed{y_t = -\frac{A}{\xi_\perp} y_{xxxx}}$$

Boundary Conditions:

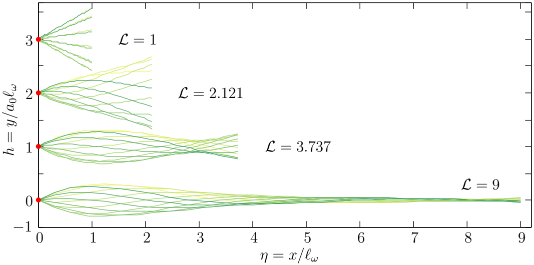


Fixed End ($x = 0$)	Free End ($x = L$)
$y = 0$	$y_{xx} = 0$
$y_x = a_0 \sin(\omega t)$	$y_{xxx} = 0$

Nondimensionalize:

$$\left.\begin{aligned} x &= L\tilde{x} \\ y &= a_0 L\tilde{y} \\ t &= \tilde{t}/\omega \end{aligned}\right\} \Rightarrow \tilde{y}_{\tilde{t}} \approx -\left(\frac{\ell_\omega}{L}\right)^4 \tilde{y}_{\tilde{x}\tilde{x}\tilde{x}\tilde{x}} \quad \text{where} \quad \boxed{\ell_\omega = \left(\frac{A}{\omega \xi_\perp}\right)^{1/4}}$$

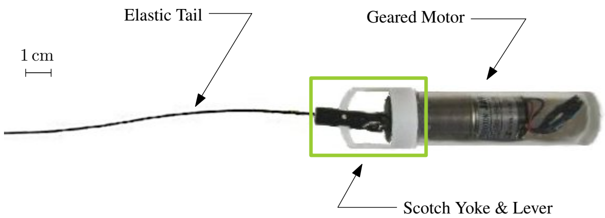
Tail shape characterized by the dimensionless length: $\boxed{\mathcal{L} = L/\ell_\omega}$



- L – tail length
- ω – oscillation frequency
- $A = EI$ – bending stiffness
- $\xi_\perp$ – transverse drag coefficient
- a_0 – angular amplitude
- $\mathcal{L}$ – dimensionless length

† Wiggins & Goldstein, *Phys. Rev. Let.*, 1998

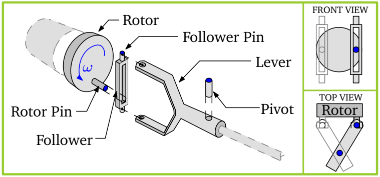
Robotic Swimmer



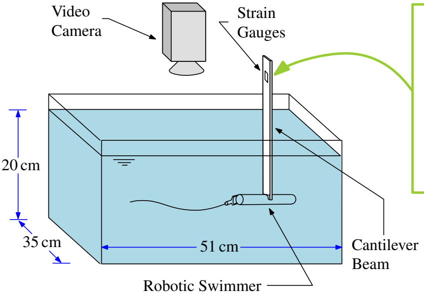
- Spring steel wires as tails
- Tail Length: $L = 18 - 30\,\text{cm}$
- Tail Diameter: $D = 0.5\,\&\,0.64\,\text{mm}$

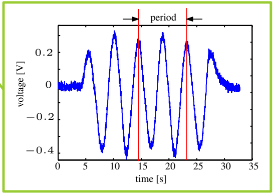
Scotch Yoke and Lever

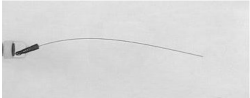
- Follower extracts horizontal component of rotor motion.
- Lever converts sinusoidal follower motion to approximately sinusoidal angle variation.



Experimental Setup

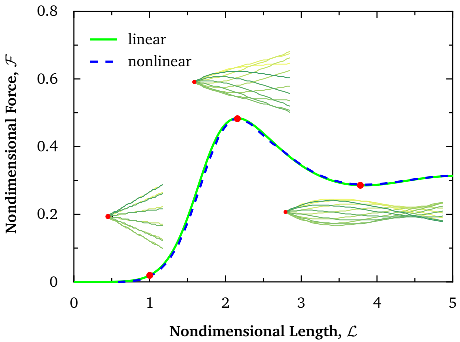






- Fluid: Silicone Oil, $\mu = 3.2\,\mathrm{Pa}\cdot\mathrm{s}$.
- Reynolds Number: $\mathrm{Re} \approx 10^{-2} - 10^{-3}$.
- Strain gages mounted on cantilever beam to measure force.
- Video camera to capture tail shapes.
- Walls close to tail.

Propulsive Force of Elastic Tail



Dimensionless Force

$$\mathcal{F} = \frac{\langle F \rangle}{{a_0}^2 {\ell_\omega}^2 (\xi_\perp - \xi_\parallel)|\omega|}$$

Dimensionless Length

$$\mathcal{L} = \frac{L}{\ell_\omega}$$

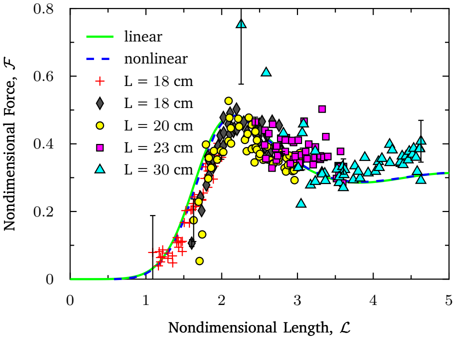
Intrinsic Length

$$\ell_\omega = \left(\frac{A}{\omega \xi_\perp}\right)^{\frac{1}{4}}$$

- Newton iteration to solve nonlinear equations.
- Maximum $\mathcal{F}$ at $\mathcal{L} \approx 2.14$

$$\langle F \rangle = \frac{1}{T}\int_0^T \int_0^L \mathbf{f}_d \cdot \hat{\mathbf{e}}_x \, dx \, dt \approx \frac{1}{T}\int_0^T \int_0^L (\xi_\parallel - \xi_\perp) y_x y_t \, dx \, dt$$
$$= a_0^2 \ell_\omega^2 (\xi_\perp - \xi_\parallel)|\omega| \left\langle y_x y_{xxx} - \frac{1}{2} y_{xx}^2 \right\rangle_{x=0}$$

Propulsive Force of Elastic Tail



Dimensionless Force

$$\mathcal{F} = \frac{\langle F \rangle}{a_0{}^2 \ell_\omega{}^2 (\xi_\perp - \xi_\parallel)|\omega|}$$

Dimensionless Length

$$\mathcal{L} = \frac{L}{\ell_\omega}$$

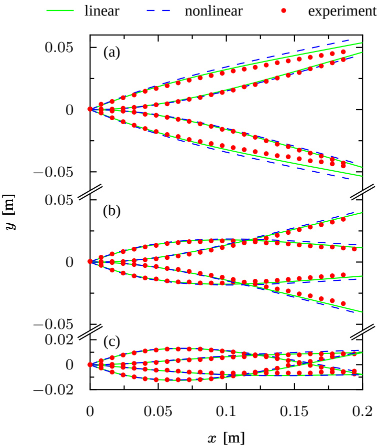
Intrinsic Length

$$\ell_\omega = \left(\frac{A}{\omega \xi_\perp} \right)^{\frac{1}{4}}$$

Data marked by $+$: $a_0 = 47^\circ, D = 0.6\,\mathrm{mm}$.
All other data: $a_0 = 25^\circ, D = 0.5\,\mathrm{mm}$.

- All parameters are known or measured.
- Drag coefficients account for wall effects.
- Actuation Frequency: $\omega = 0.06 - 0.83\,\mathrm{Hz}$.
- $L = 30\,\mathrm{cm}$ close to back wall.

Tail Shapes of Elastic Tail

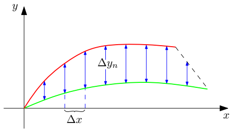


$L = 20\,\text{cm},\ D = 0.5\,\text{cm},\ a_0 = 25^\circ$

(a) $\mathcal{L} = 1.73,\quad \omega = 0.08\,\text{Hz}$

(b) $\mathcal{L} = 2.20,\quad \omega = 0.20\,\text{Hz}$

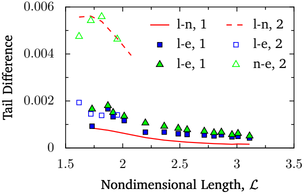
(c) $\mathcal{L} = 3.11,\quad \omega = 0.83\,\text{Hz}$



$\Downarrow$

(1) $L = 20$ cm, $D = 0.5$ mm, and $a_0 = 25^\circ$

(2) $L = 18$ cm, $D = 0.63$ mm, and $a_0 = 47^\circ$



$$\text{Tail Difference} = \frac{\|\Delta y\|}{NL}$$

$$\text{where}\quad \|\Delta y\| = \sqrt{\sum_{n=1}^{N} (\Delta y_n)^2}$$

Conclusions

Fixed Swimmer

- Oscillating, flexible tail generates propulsion at low Re.
- Linear theory describes tail motion and propulsive force well despite large actuation angle ($a_0 = 47°$).

Tony S. Yu, Eric Lauga, and A.E. Hosoi, *Phys. Fluids*, (2006)

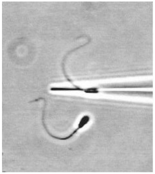
Viable propulsive mechanism?

Typical dimensions/parameters of bull spermatozoa‡:

Tail length: $L \approx 60\,\mu$m

Beat frequency: $\omega \approx 20$ Hz

Optimal elastic-tail swimmer	Bull sperm flagellum
$F \approx 70$ pN	$F \approx 250$ pN

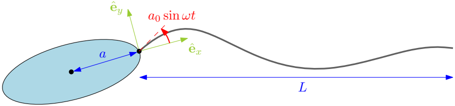


Schmitz, *et al.* (2000)

- Note: sperm have active tails.
- Passive elastic tail is simpler.
- BUT stall force not directly related to swimming velocity!

‡Brennen and Winet, *Annu. Rev. Fluid Mech.*, 1977

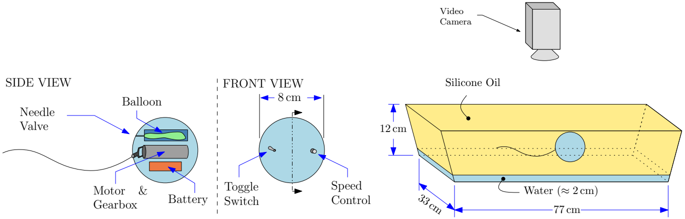
Autonomous Swimmer



Control angle relative to coordinate system that translates and rotates with body.

Torque and transverse force generated by tail balance drag of body.§

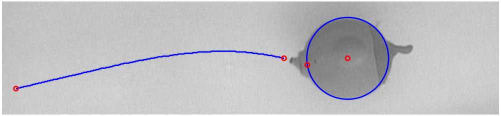
Modified Swimmer:

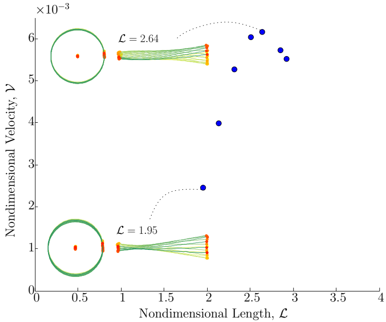


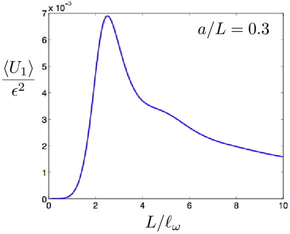
§ E. Lauga, *Phys. Rev. E*, in review

Swimming Velocity

Swimming captured and velocity analyzed:





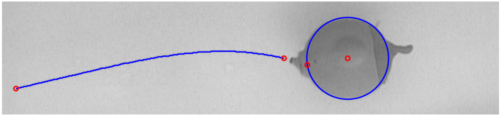


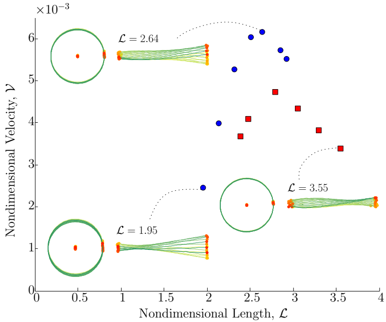
E. Lauga (in review)

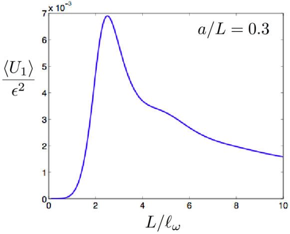
- Dimensionless Velocity: $\mathcal{V} = \dfrac{\langle U_1 \rangle}{{a_0}^2 \ell_\omega |\omega|}$
- $L = 15\,\text{cm}, \quad a = 4\,\text{cm}$

Swimming Velocity

Swimming captured and velocity analyzed:







E. Lauga (in review)

- Dimensionless Velocity: $\mathcal{V} = \dfrac{\langle U_1 \rangle}{{a_0}^2 \ell_\omega |\omega|}$
- $L = 15\,\mathrm{cm}, \quad a = 4\,\mathrm{cm}$

Summary

FIXED SWIMMER

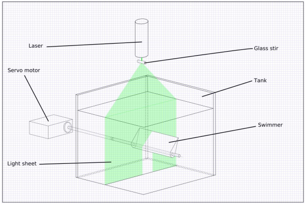
- Oscillating, flexible tail generates propulsion at low Re.
- Linear theory describes tail motion and propulsive force well despite large actuation angle ($a_0 = 47°$).

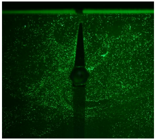
AUTONOMOUS SWIMMER

- Swimming velocity comparable to theory, but ...

Future Work

- Swimmer with multiple tails.
- Swimming in viscoelastic fluids.
- Swimming near an interface.





R. Trouillard, T.S. Yu, E. Lauga, A.E. Hosoi (2006)

Additional Slides

Hydrodynamic Drag

Wall Effects

Efficiency

Numerics

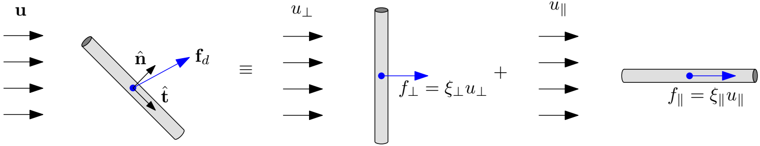
Experimental Parameters

Free Swimmer Data

Force Measurement

Image Processing

Slender-Body Hydrodynamics



Slender-body theory (Resistive-force theory) gives

$$\mathbf{f}_d = -[f_\perp \hat{\mathbf{n}} + f_\parallel \hat{\mathbf{t}}] \qquad \text{where} \quad f_\perp = \xi_\perp \mathbf{u} \cdot \hat{\mathbf{n}}, \qquad f_\parallel = \xi_\parallel \mathbf{u} \cdot \hat{\mathbf{t}}$$

$$\Rightarrow \mathbf{f}_d = -[\xi_\perp \hat{\mathbf{n}}\hat{\mathbf{n}} + \xi_\parallel \hat{\mathbf{t}}\hat{\mathbf{t}}] \cdot \mathbf{u}$$

For a slender, cylindrical rod, these drag coefficients can be expressed as

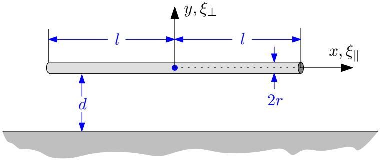
$$\xi_\perp = \frac{4\pi\mu}{\ln \frac{L}{r} + 0.193} \qquad \text{and} \qquad \xi_\parallel = \frac{2\pi\mu}{\ln \frac{L}{r} - 0.807}.$$

Drag Coefficients and Wall Effects

Unbounded fluid:

$$\xi_\perp = \frac{4\pi\mu}{\ln\frac{L}{r} + 0.193}$$

$$\xi_\parallel = \frac{2\pi\mu}{\ln\frac{L}{r} - 0.807}$$



Wall effects

$$\xi_\perp = \int_{-l}^{+l} \frac{-8\pi\mu\varepsilon}{2 + \varepsilon\{\ln(1 - x^2/l^2) + 1 - E_\perp\}} + \mathcal{O}(\varepsilon^3)dx,$$

$$\xi_\parallel = \int_{-l}^{+l} \frac{8\pi\mu\varepsilon}{4 + \varepsilon\{2\ln(1 - x^2/l^2) - 2 - E_\parallel\}} + \mathcal{O}(\varepsilon^3)dx,$$

where

$$E_\perp = \operatorname{arcsinh}\left(\frac{1+x}{2d}\right) + \operatorname{arcsinh}\left(\frac{l-x}{2d}\right) + \frac{2(l+x)}{{r_1}^{1/2}} + \frac{2(l-x)}{{r_2}^{1/2}} - \frac{(l+x)^3}{2{r_1}^{3/2}} - \frac{(l-x)^3}{2{r_2}^{3/2}}$$

$$E_\parallel = 2\operatorname{arcsinh}\left(\frac{1+x}{2d}\right) + 2\operatorname{arcsinh}\left(\frac{l-x}{2d}\right) + \frac{(l+x)}{{r_1}^{1/2}} + \frac{(l-x)}{{r_2}^{1/2}} - \frac{2(l+x)^3}{{r_1}^{3/2}} - \frac{2(l-x)^3}{{r_2}^{3/2}}$$

$$r_1 = (l+x)^2 + 4d^2, \qquad r_2 = (l-x)^2 + 4d^2 \qquad \text{and} \qquad \epsilon\ [\ln(2l/r)]^{-1}$$

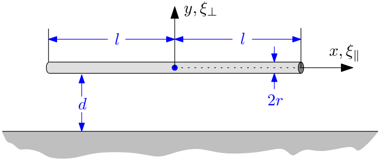
§ de Mestre and Russel, J. Eng. Math., 1975

Drag Coefficients and Wall Effects

Unbounded fluid:

$$\xi_\perp = \frac{4\pi\mu}{\ln\frac{L}{r} + 0.193}$$

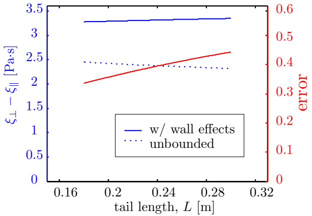
$$\xi_\parallel = \frac{2\pi\mu}{\ln\frac{L}{r} - 0.807}$$



Wall effects[¶]

$$\xi_\perp = \int_{-l}^{+l} \frac{-8\pi\mu\varepsilon}{2 + \varepsilon\{\ln(1 - x^2/l^2) + 1 - E_\perp\}} + \mathcal{O}(\varepsilon^3)dx,$$

$$\xi_\parallel = \int_{-l}^{+l} \frac{8\pi\mu\varepsilon}{4 + \varepsilon\{2\ln(1 - x^2/l^2) - 2 - E_\parallel\}} + \mathcal{O}(\varepsilon^3)dx,$$



where

$$E_\perp = \operatorname{arcsinh}\left(\frac{1+x}{2d}\right) + \operatorname{arcsinh}\left(\frac{l-x}{2d}\right) + \frac{2(l+x)}{{r_1}^{1/2}} + \frac{2(l-x)}{{r_2}^{1/2}} - \frac{(l+x)^3}{2{r_1}^{3/2}} - \frac{(l-x)^3}{2{r_2}^{3/2}}$$

$$E_\parallel = 2\operatorname{arcsinh}\left(\frac{1+x}{2d}\right) + 2\operatorname{arcsinh}\left(\frac{l-x}{2d}\right) + \frac{(l+x)}{{r_1}^{1/2}} + \frac{(l-x)}{{r_2}^{1/2}} - \frac{2(l+x)^3}{{r_1}^{3/2}} - \frac{2(l-x)^3}{{r_2}^{3/2}}$$

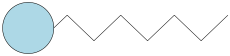
$$r_1 = (l+x)^2 + 4d^2, \qquad r_2 = (l-x)^2 + 4d^2 \qquad \text{and} \qquad \epsilon\,[\ln(2l/r)]^{-1}$$

[¶] de Mestre and Russel, J. Eng. Math., 1975

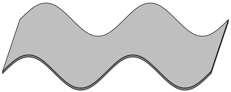
Efficiency

The efficiency of swimming at low Reynolds number can be defined as (from Childress):

$$\eta = \frac{F_p \cdot U}{\int \mathbf{f} \cdot \mathbf{u} \, ds}$$



$\eta \approx 8.6\%$ from Lighthill (1976)

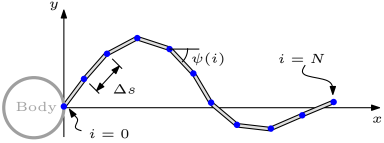


$\eta \approx 7\%$ from Taylor (1951)

$\eta \approx 0.8\%$ from Purcell (1997)

$\eta \approx 0.1\%$ from Lauga (in Review)

Numerical Solution of Nonlinear Equations



- Newton-Raphson iteration.
- Break tail up into N segments of length Δs.
- Change derivatives to difference equations.

$$\frac{\psi_i^{n+1} - \psi_i^n}{\Delta t} = -\frac{A}{\xi_\perp}\frac{\phi_{i+1}^{n+1} - 2\phi_i^{n+1} + \phi_{i-1}^{n+1}}{\Delta s^2} \quad \text{where} \quad \phi_i = \frac{\psi_{i+1} - 2\psi_i + \psi_{i-1}}{\Delta s^2}$$

$$\underbrace{\begin{bmatrix} f_{1,1} \\ f_{2,1} \\ \vdots \\ f_{1,N} \\ f_{2,N} \end{bmatrix}}_{\mathbf{f}} + \underbrace{\begin{bmatrix} \frac{\partial f_{1,1}}{\partial \psi_1} & \frac{\partial f_{1,1}}{\partial \phi_1} & \cdots & \frac{\partial f_{1,1}}{\partial \psi_N} & \frac{\partial f_{1,1}}{\partial \phi_N} \\ \frac{\partial f_{2,1}}{\partial \psi_1} & \frac{\partial f_{2,1}}{\partial \phi_1} & \cdots & \frac{\partial f_{2,1}}{\partial \psi_N} & \frac{\partial f_{2,1}}{\partial \phi_N} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{\partial f_{1,N}}{\partial \psi_1} & \frac{\partial f_{1,N}}{\partial \phi_1} & \cdots & \frac{\partial f_{1,N}}{\partial \psi_N} & \frac{\partial f_{1,N}}{\partial \phi_N} \\ \frac{\partial f_{2,N}}{\partial \psi_1} & \frac{\partial f_{2,N}}{\partial \phi_1} & \cdots & \frac{\partial f_{2,N}}{\partial \psi_N} & \frac{\partial f_{2,N}}{\partial \phi_N} \end{bmatrix}}_{\mathbf{J}} \underbrace{\begin{bmatrix} \Delta\psi_1 \\ \Delta\phi_1 \\ \vdots \\ \Delta\psi_N \\ \Delta\phi_N \end{bmatrix}}_{\Delta\mathbf{x}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}.$$

Experimental Parameters

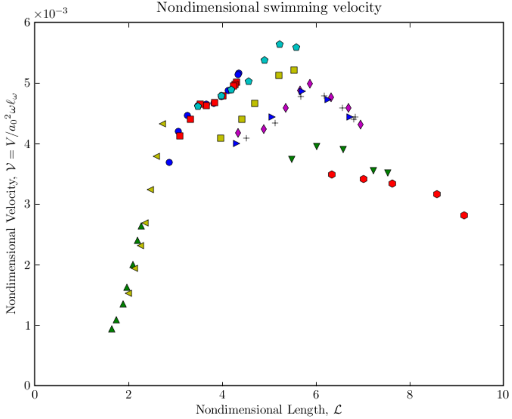
Tail Length L[m]	Tail Diameter D[mm]	Young's Modulus E[GPa]	Moment of Inertia I [m^4] $\times 10^{15}$	Bending Stiffness A [N $\cdot$ m^2] $\times 10^3$
0.18–0.3	0.51 & 0.61	190	3.3 & 6.8	0.62 & 1.3

Fluid Viscosity μ[Pa $\cdot$ s]	Oscillation Frequency ω[rad/s]	Angular Amplitude a_0[rad]	Reynolds Number Re
3.18	0.4–5	0.814 & 0.435	10^{-2}–10^{-3}

Young's Modulus E [GPa]	Beam Length L [m]	Beam Thickness h [cm]	Beam Width b [cm]	Moment of Inertia I [m^4]	Strain Gage Position d [cm]
3.1	0.26	0.297	1.88	4.1×10^{-11}	1.27

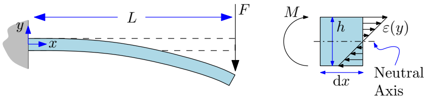
Input Voltage V_{in} [V]	Gage Factor GF	Circuit Resistances R [Ω]	R_5 [kΩ]	R_6 [kΩ]	R_7 [kΩ]	R_8 [kΩ]
5	2.13	120	10	500	10	1000

Free Swimming Data

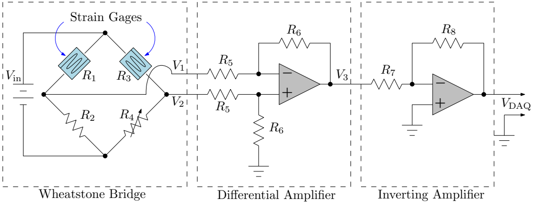


- Dimensionless Velocity: $\mathcal{V} = \dfrac{\langle U_1 \rangle}{{a_0}^2 \ell_\omega |\omega|}$
- $L = 30\,\mathrm{cm}, \quad a = 4\,\mathrm{cm}$

Force Measurement



$$\left.\begin{aligned} M &= -F(L-x) \\ \sigma &= -\frac{My}{I} \end{aligned}\right\} \varepsilon = \frac{F(L-x)y}{EI}$$

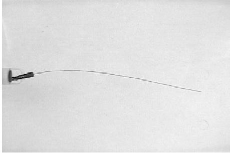


$$(V_1 - V_2) = V_{in}\left[\frac{R_1 R_2}{(R_1+R_2)^2}\left(\frac{dR_1}{R_1}\right) + \frac{R_3 R_4}{(R_3+R_4)^2}\left(-\frac{dR_3}{R_3}\right)\right]$$

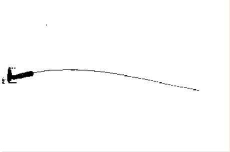
$$V_3 = \frac{R_6}{R_5}(V_1 - V_2)$$

$$V_{DAQ} = -\frac{R_8}{R_7} V_3 .$$

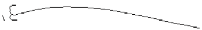
Image Processing Sequence



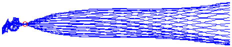
(a) Grayscale image from video.



(b) Black and white image after thresholding grayscale image.



(c) Black and white image after filtering operations.



(d) Overlay of tail shape data at successive time intervals.

User Interface for Image Processing